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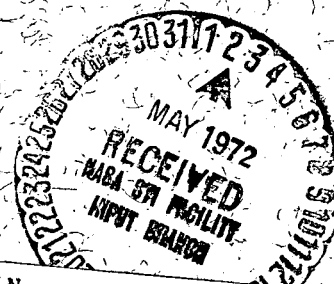
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STATIONARY COLLECTOR IN A COLLISIONLESS PLASMA

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STATIONARY COLLECTOR IN A COLLISIONLESS PLASMA

by

J. M. Grebowsky

ABSTRACT

A neutral collecting body in a collisionless plasma can modify the plasma distribution in its immediate neighborhood. When a magnetic field is present an electron plasma with a neutralizing background charge has empty velocity space regions at points near a collector even though the distribution is Maxwellian far from the collector. For a thin cylinder, the collected collisionless plasma current is a function of the angle between the cylinder axis and the magnetic field with the minimum current collected when the cylinder and field lines are parallel.

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I. INTRODUCTION

When considering the flux of charged particles intercepted by a stationary surface of finite dimensions in a magnetic field free plasma, isotropy can be assumed even when the collisional mean free path is much greater than the dimensions of the collecting body. As particles collide with and are absorbed by the collecting body they are replaced by particles with the same velocities produced by randomizing collisions far from the collecting surface. Hence in the immediate vicinity of the absorbing surface all regions of velocity space which can be collected are fully occupied.

If however a magnetic field permeates the space about a collecting body, the transport of plasma across the field lines is inhibited compared to the transport along the field lines if the collision frequency is smaller than the average frequency of gyration of the charged particles about the field line. Hence in a collisionless magnetic plasma (i.e., collisional mean free path much greater than the average gyroradius) gyrating particles collected by the collecting surface are replenished only by drifting along the field lines from outside the region of the collector as they gyrate about the field with fixed gyroradii. This asymmetrical replenishment process in a magnetic field can result in a reduced collected current compared to the current incident in a field free plasma. Also as will be seen the collected current can vary as the orientation of the collecting body changes with respect to the magnetic field direction.

II. DENSITY PERTURBATION

The effect a magnetic field has on the plasma flux incident upon a stationary collecting surface in a collisionless plasma can be demonstrated using as an example a cylindrical collecting body of finite length placed in a uniform magnetic field. The radius of the cylinder for simplicity is chosen to be much smaller than the average gyroradius. It is assumed that a collisionless plasma (i.e., mean free path much greater than other characteristic lengths) of electrons with a neutralizing background charge populates the space outside the cylinder. The interparticle interactions, although infrequent, will produce a Maxwellian distribution of electrons at large distances from the cylinder. Also to avoid the complexities resulting from electric field accelerations the net charge on the cylinder surface is taken as zero.

Consider first the case when the cylinder axis is oriented parallel to the magnetic field. For this alignment the use of cylindrical coordinates is appropriate with the z axis coincident with the cylinder axis and pointed in the direction of the uniform magnetic field. Denoting the length of the cylinder by L and its radius by a , one end of the cylinder is fixed at $z = 0$ and the other at $z = L$. Without collisions an electron will travel with a constant velocity component v_{\parallel} in the field direction as it gyrates about a field line with radius of gyration $r_g = mv_{\perp} / eB$ and period $t_g = 2\pi m / eB$ and m is the mass and e the charge of the electron, B is the magnitude of the field and v_{\perp} is the gyration speed of the electron.

An electron moving in the direction of increasing z (i.e., to the right in Figure 1) while gyrating about a field line can hit the collecting surface only if the center of gyration is located approximately one gyroradius from the collector – the probe radius is assumed to be much smaller than the average gyroradius. Consider such an electron moving along a trajectory which passes through $z = 0$ in the direction of increasing z . If the electron is on the collector axis at $z = 0$ and is moving in the direction of the field with parallel velocity v_{\parallel} then it will gyrate from its initial polar radius $r = 0$ at $z = 0$ to its maximum distance $r = 2r_g$ from the axis at $z = v_{\parallel} t_g / 2$ and then back to the cylinder axis $r = 0$ at $z = v_{\parallel} t_g$ where it is collected. All collectable electrons with the same v_{\parallel} but with trajectories for which $r \neq 0$ at $z = 0$ will strike the cylinder at z values less than z/t_g . Hence only those electrons with positive parallel velocities greater than z/t_g reach the cylinder at z .

In a similar manner electrons moving in the negative z direction along the length of the cylinder will originate in the region $z \geq L$ and will exist on the surface of the cylinder at z only if the parallel velocity component is less than $(z-L)/t_g$. Hence when each electron incident upon the cylinder is collected the number density at the cylinder surface is from Liouville's theorem:

$$n = \pi \int_0^{\infty} \left(\int_{z/t_g}^{\infty} f dv_{\parallel} + \int_{-\infty}^{(z-L)/t_g} f dv_{\parallel} \right) v_{\perp} dv_{\perp} \quad (1)$$

where f is the Maxwellian distribution function which characterizes the electron distribution at large distances from the cylinder:

$$f = n_0 \left(\frac{m}{2\pi kT} \right)^{3/2} e^{-\frac{1}{2} m v^2 / kT} \quad (2)$$

where $v^2 = v_{\parallel}^2 + v_{\perp}^2$, n_0 is the ambient number density far from the cylinder and T is the temperature. The factor of π in (1) results from integration over all the allowable angles of incidence at a point on the cylinder's surface. Only those electrons with gyration velocities directed inward across the surface have not been previously collected in their motion along the field lines.

Using (2) the density distribution along the cylinder is

$$n = \frac{n_0}{2} \left[1 - \frac{1}{2} \operatorname{erf} \left(\sqrt{\frac{m}{2kT}} \frac{z}{t_g} \right) - \frac{1}{2} \operatorname{erf} \left(\sqrt{\frac{m}{2kT}} \frac{L-z}{t_g} \right) \right]. \quad (3)$$

Since the error function approaches unity when its argument becomes large, the density in the immediate neighborhood of the cylinder will on the average be much smaller than the ambient density n_0 when the average thermal speed of the electron distribution is much smaller than L/t_g .

If on the other hand the cylinder axis is aligned perpendicular to the direction of the magnetic field, the gyrating electrons which strike the cylinder traverse only a short field aligned collection region of length less than or equal to the cylinder diameter. If the cylinder radius a is small compared to the average

gyroradius as assumed, then $t_g \sqrt{2kT/m}$ will be much larger than a and by inference from the previous calculation essentially all regions of velocity space are occupied by electrons at the cylinder surface (of course, the phase of the perpendicular velocity must still be restricted to the inward direction at the surface). Hence the density at all points on the cylinder will be just $n_0/2$ which also characterizes the density expected in the absence of a magnetic field. This density is greater than that near the cylinder in the parallel configuration.

III. CURRENT COLLECTION

Although the density perturbation induced by the collecting cylinder is a vivid example of the asymmetry associated with a magnetic field in a collisionless plasma it is not the density distribution along the collector which is usually directly measured. Indeed, usually the total current intercepted by a collector (called a probe) is measured and analytical expressions relating the current to the properties of the ambient plasma are inverted to obtain the characteristic plasma parameters.^{1,2} This current will also show the effects of the collection asymmetry due to the magnetic field.

Again consider the thin cylinder of finite length when its axis is parallel to the magnetic field lines. The regions of velocity space occupied by the electrons at the surface of the cylinder were previously determined so that the net flux of thermal electrons incident normally at a point on the cylinder is:

$$J = \int_{-\pi/2}^{\pi/2} \int_0^{\infty} v_{\perp} \cos \varphi \left[\int_{z/t_g}^{\infty} f dv + \int_{-\infty}^{(z-L)/t_g} f dv \right] v_{\perp} dv_{\perp} d\varphi \quad (4)$$

where φ is the angle between the velocity of gyration and the inward normal to the cylinder surface and f is Maxwell's distribution function.

Performing the integration it is found that the flux incident upon the cylindrical surface is just the Knudsen diffusion flux³ $\frac{n_0}{4} (8\pi kT/m)^{1/2}$ modified by the factor depicting the density depression due to the existence of depleted velocity space regions:

$$J = \frac{n_0}{4} \left(\frac{8\pi kT}{m} \right)^{1/2} \left[1 - \frac{1}{2} \operatorname{erf} \left(\sqrt{\frac{m}{2kT}} \frac{z}{t_g} \right) - \frac{1}{2} \operatorname{erf} \left(\sqrt{\frac{m}{2kT}} \frac{L-z}{t_g} \right) \right] \quad (5)$$

The total current intercepted by the cylindrical surface is therefore

$$I = \int_0^L 2\pi r J dz$$

or

$$I = \pi a n_0 \left(\frac{2kT}{\pi m} \right)^{1/2} \left[L - L \operatorname{erf} \left(\sqrt{\frac{m}{2kT}} \frac{L}{t_g} \right) - \frac{t_g}{\sqrt{\frac{\pi m}{2kT}}} \left(e^{-\frac{m}{2kT} \frac{L^2}{t_g^2}} - 1 \right) \right] \quad (6)$$

Another contribution to the net current results from plasma incident upon each of the flat ends of the cylinder. This component is just the product of the Knudsen diffusion flux and the area of the end surface so that the total current collected when the cylinder is aligned along the direction of the magnetic field is:

$$I_{\parallel} = \pi a n_0 \left(\frac{2kT}{\pi m} \right)^{1/2} \left[a + L \left\{ 1 - \operatorname{erf} \left(\sqrt{\frac{m}{2kT}} \frac{L}{t_g} \right) - \frac{\sqrt{\frac{2kT}{m}} t_g}{L} \left(e^{-\frac{m}{2kT} \frac{L^2}{t_g^2}} - 1 \right) \right\} \right] \quad (7)$$

Hence the total current decreases towards its minimum value

$$\pi a n_0 \left(\frac{2kT}{\pi m} \right)^{1/2} \left[a + \sqrt{\frac{2kT}{m}} t_g \right]$$

when the length of the cylinder becomes much greater than the average distance an electron travels in one gyroperiod t_g .

On the other hand, when the cylinder axis is perpendicular to the magnetic field the depleted velocity space regions are not significant and the expression for the net incident current is identical to that describing the plasma current in a field free plasma:

$$I_{\perp} = \pi a n_0 \left(\frac{2kT}{\pi m} \right)^{1/2} (L + a). \quad (8)$$

This current incidentally is also collected in the parallel configuration described by (7) when the cylinder length is very small compared to the average distance the electrons travel in one gyroperiod.

Hence a significant difference can exist between the currents incident upon the cylinder oriented at different angles with respect to the magnetic field in a collisionless plasma. The extent of this asymmetry is dependent upon the relative scale of the charged particle gyrations in comparison to the length of the cylinder.

IV. CONCLUSIONS

Although only a stationary cylindrical body was considered explicitly, other collecting configurations will also introduce perturbations in the plasma distribution within their range of influence due to the effect of the magnetic field on the charged particle trajectories. Hence in a magnetic field the current collected will be controlled by the relative direction of the field regardless of the collector shape with the magnitude of the current less than or equal to that in the field free case.

The interaction between a collecting body and a collisionless plasma, although interesting in itself, is of practical importance. Properties of the ionized medium above the earth's surface are measured using current collecting probes. At high altitudes where the collisionless approximation is valid and the plasma motion is dominated by the earth's magnetic field, such investigations using cylindrically shaped probes⁴ have observed a modulation in the collected current as the probe changes orientation with respect to the magnetic field. From this observation and the dynamics discussed in this paper it is evident that the perturbation of the ambient collisionless plasma by the collecting body must be taken into account in order to interpret the measurements.

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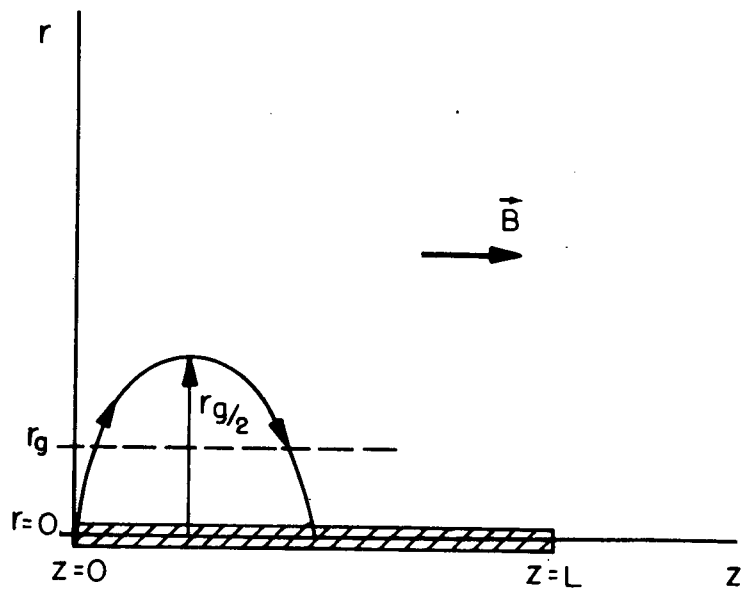


Figure 1. Cylinder aligned parallel to magnetic field